

## II. *On the Specific Resistance of Mercury.*

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OUR experiments on the determination of the British Association unit of electrical resistance in absolute measure are detailed in two memoirs communicated to the Society.\* The conclusion to which they led us is that

$$1 \text{ B.A. unit} = \cdot 9865 \frac{\text{earth quadrant}}{\text{second}},$$

but this result differs considerably from that obtained by some other experimenters, the original Committee included. Although in the present state of the question it is not desirable that the B.A. unit should fall into disuse, there can be no question as to the importance of connecting it with the mercury unit introduced now more than twenty years ago by SIEMENS. It will then be possible, as recommended by the Paris Conference, to express our absolute measurements in terms of mercury, by stating what length of a column of mercury at 0° of 1 square millimetre section has a resistance of 1 ohm. Accordingly the experiments about to be described relate to the expression in terms of the B.A. unit of the resistances of known columns of mercury at 0°.

This investigation was the more necessary, as the principal authorities on the subject, Dr. WERNER SIEMENS and Dr. MATTHIESSEN, had obtained results differing by as much as ·8 per cent.

The earlier determinations of SIEMENS were vitiated by the assumption of an erroneous value (13·557) for the specific gravity of mercury, a constant which it is necessary to know in order to infer the mean section of a tube from the weight of contained mercury. The error, pointed out by MATTHIESSEN, was afterwards† admitted by SIEMENS, who gives as the corrected expression of the relation between the two units,

$$1 \text{ mercury unit} = \cdot 9536 \text{ B.A. unit.}$$

On the other hand, the independent measurements of the resistance of mercury by MATTHIESSEN and HOCKIN‡ gave

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† Phil. Mag., xxxi., 1866.

‡ Reprint of British Association Reports, p. 114.

1 mercury unit = .9619 B.A. unit,

the mercury unit being defined as the resistance at  $0^\circ$  of a column of mercury 1 metre long and 1 square millimetre in section.

Our own experiments lead us to a value not differing much from that of SIEMENS. We find

1 mercury unit = .95418 B.A. unit.

If we assume that the B.A. unit is .98651 ohm (in accordance with our determination), we find

1 mercury unit = .94130 ohm,

the ohm being  $10^9$  C.G.S. The same result may be expressed in another way by saying that the ohm is the resistance of a column of mercury at  $0^\circ$ , 1 square millimetre in section, and 1062.4 millims. in length.

Through the kindness of Dr. C. W. SIEMENS we have had an opportunity of comparing with the B.A. units a standard mercury unit (No. 2513) issued by Messrs. SIEMENS and HALSKE. At the proper temperature ( $16^\circ.7$ ) we find that its resistance is

.95365 B.A. unit,

agreeing very closely with previous comparisons of SIEMENS' mercury measurements with the B.A. unit.

The determination of the specific resistance of mercury is simple enough in principle, though the execution is somewhat tedious, and the calculation of the results is complicated in practice by the necessity of introducing various temperature corrections. In a first sketch of the method it will be convenient to omit these corrections, which is tantamount to supposing that all the measurements are made at zero. If  $L$  be the length and  $s$  the section of the column of mercury,  $R$  its resistance,  $r$  the specific resistance of the metal,

$$R = \frac{rL}{s}, \quad \text{or} \quad r = R \frac{s}{L}$$

The length  $L$  can be measured directly, but  $s$  can only be found with the necessary accuracy from the contents. Thus if  $\rho$  be the specific gravity of mercury, and  $W$  the weight of the whole column in grammes,  $\rho Ls = W$ , whence  $s = W/\rho L$ , and

$$r = \frac{RW}{\rho L^2}$$

Apart from the temperature corrections already referred to, the simplicity of the formula is disturbed by the inevitable departure from the truly cylindrical form of the glass tubes used to contain the mercury. It is true indeed that to a first order of approximation the formula stands unaltered, as we may see if we understand by  $s$  the *mean* section of the tube. The volume is still truly expressed by  $sL$ , and the resistance is *approximately* expressed by  $rL/s$ . If, however, the squares of the variations

of section cannot be neglected, the actual resistance is greater than the formula would lead us to suppose, as is evident if we imagine the section to become at one place very small.

In general we must regard  $s$  as a function of the position ( $x$ ) along the tube at which it is taken. For the purposes of the present paper we may assume with sufficient approximation (see Lord RAYLEIGH'S 'Theory of Sound,' § 308)

$$R = r \int \frac{dx}{s}$$

The necessary data with respect to  $s$  are obtained by a calibration of the tube. "If a small quantity of mercury is introduced into the tube and occupies a length  $\lambda$  of the tube, the middle point of which is distant  $x$  from one end of the tube, then the area  $s$  of the section near this point will be  $s = C/\lambda$ , where  $C$  is some constant. The weight of mercury which fills the whole tube is

$$W = \rho \int s dx = \rho C \Sigma \left( \frac{1}{\lambda} \right) \frac{L}{n}$$

where  $n$  is the number of points at equal distances along the tube, where  $\lambda$  has been measured, and  $\rho$  is the mass of unit of volume.

"The resistance of the whole tube is

$$R = \int \frac{r dx}{s} = \frac{r}{C} \Sigma (\lambda) \frac{L}{n}$$

"Hence

$$WR = r \rho \Sigma (\lambda) \Sigma \left( \frac{1}{\lambda} \right) \frac{L^2}{n^2}$$

and

$$r = \frac{WR}{\rho L^2} \frac{n^2}{\Sigma (\lambda) \Sigma \left( \frac{1}{\lambda} \right)}$$

gives the specific resistance of unit of volume" (MAXWELL'S 'Electricity,' § 362).

In the sequel

$$\frac{1}{n^2} \Sigma (\lambda) \Sigma \left( \frac{1}{\lambda} \right)$$

is denoted by  $\mu$ ; it is a numerical quantity a little greater than unity.

Another correction is required in our method of working to take account of the resistance offered by that part of the mercury in the terminal cups, which is situated just beyond the ends of the tube. The question is identical with that of the correction necessary in calculations of pitch for the open ends of organ pipes (see 'Theory of Sound,' § 307, and Appendix A), and it scarcely admits of absolutely definite solution. We cannot, however, be far wrong in adding to the actual length of the tube  $\cdot 82$  of its diameter, which corresponds to the supposition that the diameter of the mercury column suddenly becomes infinite. Since, in our experiments, the whole correction

only amounts to about a thousandth part, even a ten per cent. error in our estimate would scarcely be material.

Let  $r$  = resistance of a column of mercury 1 metre long and 1 square millimetre in section, at  $0^\circ$ , expressed in B.A. units.

$R$  = resistance of the tube full of mercury at  $0^\circ$  in B.A. units.

$L$  = length of the tube at  $t^\circ$  in centimetres as measured with brass rod.

$l$  = length of a thread of mercury of nearly the length of the tube at  $t^\circ$  as measured with brass rod.

$W$  = weight of the same thread in grammes.

$\mu$  = coefficient correcting for conicality of tube.

$\delta L$  = correction to  $L$  on account of the connecting rods not being close up to the ends of the tube =  $\cdot 82 \times$  diameter of tube.

$\rho$  = specific gravity of mercury at  $0^\circ = 13\cdot 595$ .

$\gamma$  = cubic expansion of mercury per degree =  $\cdot 0001795$ .

$g$  = " " glass " =  $\cdot 000025$ .

$b$  = linear expansion of brass " =  $\cdot 000018$ .

$t_0$  = temperature of brass measuring rod to which the lengths are corrected =  $17^\circ\cdot 2$ .

Then the volume of the thread at  $0^\circ = W/\rho$

$$\text{,, ,, } t^\circ = \frac{W}{\rho}(1 + \gamma t)$$

$$\text{Mean section of the tube at } t^\circ = \frac{W(1 + \gamma t)}{\rho l \{1 + b(t - t_0)\}}$$

$$\text{Mean section at } 0^\circ = \frac{W(1 + \gamma t)}{\rho l \{1 + b(t - t_0)\} \{1 + \frac{2}{3}gt\}}$$

$$\text{Length of the tube at } 0^\circ = \frac{(L + \delta L) \{1 + b(t' - t_0)\}}{1 + \frac{1}{3}gt'}$$

$$R = 10^{-4} \cdot r \cdot \mu \cdot \frac{(L + \delta L) \{1 + b(t' - t_0)\}}{1 + \frac{1}{3}gt'} \cdot \frac{\rho l \{1 + b(t - t_0)\} \{1 + \frac{2}{3}gt\}}{W(1 + \gamma t)}$$

$$r = \frac{10^4 RW(1 + \gamma t)(1 + \frac{1}{3}gt')}{\rho \mu l L(1 + \frac{2}{3}gt)} \left( 1 - \frac{\delta L}{L} \right) \{ 1 - b(t + t' - 2t_0) \}$$

The value of  $\rho$  is that used by the Committee of the British Association in reducing Dr. MATTHIESSEN'S experiments (see reprint of 'Reports on Electrical Standards,' p. 114), and stated to be the mean of the values given by KOPP, REGNAULT, and BALFOUR STEWART. The values of  $g$ ,  $\gamma$ , and  $b$  are taken from EVERETT'S 'Units and Physical Constants'— $\gamma$  being REGNAULT'S value for the expansion of mercury. The measurements of the other quantities, which depend on the particular tube used, are given in the following table, together with the resulting value of  $r$ . The description of the means employed to obtain these data follows.

Number of observation.	Date of observation, 1882.	Number of tube.	R.	Temperature of coil F.	Temperature of second coil.	L in centi-metres.	$\mu$ .	$l$ in centi-metres.	W.	$t$ .	$t'$ .	$\frac{.955 \times \delta L}{L}$ .	$\tau b(t + t' - 2t_0)$ .	$\tau$ .	Mean values of $\tau$ from each tube.
1	Feb. 23 & 24	I.	.79912	{ 12.2 11.5	12.1 } 11.5 }	87.771	1.00314	87.234	12.442	16.5	16.5	.00103	+ .00002	.95386	.95416
2	" 25	I.	.79912	12.7	12.5	"	"	87.310	12.4545	17.2	17.2	"	.00000	.95412	
3	" 21	I.	"	"	"	"	"	87.035	12.4185	18.4	"	"	-.00002	.95424	
4	March 18	I.	.79920	13.7	13.75	"	"	87.558	12.436	20.6	"	"	-.00006	.95436	
5	Weighed Feb. 14	I.	.79912	"	"	"	"	87.771	12.523	16.5	16.5	"	+ .00002	.95421	
6	Feb. 24	II.	.99088	12.0	"	96.400	1.00007	96.054	12.096	16.7	16.4	.000883	+ .00002	.95389	.95419
7	" 21 to 23	II.	.99081	13.2	"	"	"	95.432	12.0245	16.4	"	"	+ .00003	.95414	
8	March 7	II.	.99081	11.5	"	"	"	95.831	12.074	17.1	"	"	+ .00002	.95437	
9	" 8	II.	.99079	12.2	"	"	"	96.151	12.113	18.0	18.0	"	-.00003	.95436	
10	" 30	II.	.99085	13.25	"	"	"	"	"	"	"	"	"	"	
11	" 6	III.	.99711	11.2	"	123.566	1.00046	122.218	19.620	16.2	18.7	.000778	-.00001	.95424	.95416
12	" 10	III.	.99725	12.9	"	"	"	123.288	19.780	18.5	"	"	-.00005	.95418	
13	" 13	III.	.99720	12.7	"	"	"	123.221	19.7665	18.4	"	"	-.00005	.95399	
14	" 14	III.	.99725	13.4	"	"	"	123.058	19.745	18.3	"	"	-.00005	.95425	
15	" 22	IV.	.50783	13.0	12.9	194.137	1.000838	193.410	95.859	14.5	14.5	.000869	+ .00009	.95440	
16	" 24	IV.	.50774	12.7	12.7	"	"	192.576	95.402	16.8	"	"	+ .00005	.95415	

Mean of all the above values of  $\tau$  in B.A. units .95418.

The mercury used for all the measurements except 10 and 14 was distilled in vacuo with an apparatus fitted up by Mr. SHAW. In order to see whether a different result might not be obtained with other mercury, some was procured from the chemical laboratory for measurements 10 and 14. For the latter a portion of this mercury was treated with nitric acid and distilled at atmospheric pressure. For measurement 10 it was treated with nitric acid, but not distilled. An accident occurred in carrying out this measurement, so that only the resistance of the column was ascertained; but this agrees so well with the resistances found with the same tube for the other mercury, that there is no reason to suppose that any discrepancy would have appeared in proceeding with the measurement further.

The glass tubes used were supplied by CASSELLA, and were selected for uniformity of bore, so that the correction for conicality should be small. They were slender and easily broken, which made the manipulation of them difficult, and it was in fact owing to a breakage that the tube called No. I. was used so short. The measurements taken with it, at first intended to be preliminary, were, however, made with the same care as in the case of the other tubes, and the difference of length and resistance adds some variety to the data. Tubes II. and III. were cut so that their resistance should be as nearly as possible one B.A. unit. The section of tubes I., II., and III., was approximately 1 square millimetre. Tube IV. was a much larger one, introduced with a view of varying the data as much as could conveniently be done. The diameter of its bore was about 2 millims., and its length was nearly 2 metres. It was cut so as to give a resistance of about half a B.A. unit.

The ends of the tubes were ground into a convex form with emery powder on a lathe, in order that the length ( $L$ ) of the bore might be measured accurately. This measurement was effected by setting two microscopes, which could be adjusted longitudinally to the exact position required by micrometer-screws graduated to  $\frac{1}{10000}$  inch, so that their cross-wires should coincide with the ends of the tube. Observations were made in three or four different positions as the tube was turned round its axis, and the mean taken. After removal of the tube, a brass measuring rod belonging to the British Association was substituted for it, and the number of whole divisions corresponding most nearly to the distance between the cross-wires of the two microscopes was read off. The outstanding fraction of a millimetre was then ascertained by screwing the microscope up to the whole division and reading the difference on the screw-head. For the long tube the measuring rod was too short, and a third microscope had to be used to fix an intermediate point as a fresh departure for the scale. A thermometer laid beside the tube during the measurement gave the temperature ( $t'$ ) at the moment. The brass measuring rod was carefully examined, and its divisions were found to agree among themselves.

The tubes were cleaned by passing through them in succession, by means of a suction-pump, sulphuric acid, nitric acid, caustic potash, and distilled water, followed by air dried with chloride of calcium. The process with omission of the acids was in

general repeated between each refilling with mercury, but it was omitted in measurement 7, and there is no record of its having been done in 1, 3, and 6.

To calibrate the tubes a short thread of mercury was inserted and moved to the various positions required, by blowing through a chloride of calcium tube. In the case of tubes I. and II., the length,  $\lambda$ , of the thread was measured by adjusting microscopes to its two ends, with subsequent substitution of an ivory scale divided in fiftieths of an inch. But this method was troublesome; and with tubes III. and IV. the scale was simply placed against the thread and the length read off with a magnifying-glass, a procedure which was found to give sufficiently accurate results, notwithstanding the difficulty arising from parallax owing to the thickness of the glass. The following table gives the different values of  $\lambda$  for each tube.

As a check upon the correction for conicality, two distinct values of  $\mu$  were in some cases calculated from the alternate observations of  $\lambda$ , and were found to agree closely. It may not be superfluous to mention that in carrying out the computations we must work to six or seven places, although the observed values of  $\lambda$  themselves may not be accurate beyond the third place.

The lengths are in fiftieths of an inch.			
Tube I.	Tube II.	Tube III.	Tube IV.
80·8	104·5	135·0	171·0
80·0	104·1	134·0	172·0
77·0	104·5	133·0	171·5
75·8	105·0	132·0	170·5
76·0	104·5	131·5	171·5
76·4	105·2	130·5	174·5
75·0	104·3	128·0	175·0
74·0	104·0	127·5	174·5
73·4	104·7	126·5	175·5
73·0	104·0	126·5	176·5
72·7	103·0	126·5	177·0
72·3	101·8	126·0	180·0
72·5		125·0	180·5
71·9		125·5	180·7
71·1		126·0	182·2
70·1		126·0	183·7
69·7		126·0	183·5
68·0		126·5	182·5
67·9		127·0	184·0
67·6		127·0	186·0
65·9		128·5	186·5
65·3		128·0	
		128·5	
		128·0	

To find the mean section of the tubes we at first tried the method adopted by Messrs. MATTHIESSEN and HOCKIN in their experiments for the British Association. After aspirating the tube with dry air we placed it in a wooden trough full of mer-

cury, and filled it by suction. It was then held down in the trough with iron weights till it was presumably of the same temperature as the mercury in the trough, which was taken at three places. It was then held by the fingers (previously cooled in other mercury), pressed against its two ends, and taken out of the trough, the mercury adhering to the outside was brushed off, and the contents of the tube were emptied into a small porcelain crucible and weighed. But there was no doubt that when the fingers holding the tube were bare they pressed a little way—how much it was difficult to determine—into the tube, and when they were covered with stiff leather, or other stiff material, it was difficult to get a sufficiently good hold. However, in one case (No. 5)  $r$  was calculated from the weight so obtained with leather on the fingers.

The method, followed by SIEMENS and SABINE, of screwing an iron plate up against the end of the tube, was attempted, but we did not succeed in closing the orifice sufficiently tightly in this way. Ultimately we came to the conclusion that the best results would be obtained by weighing a thread of mercury nearly as long as the tube, and of which we could ascertain the actual length by direct measurement. We thought, also, that there might be some advantage in ascertaining the volume of the mercury from the same filling as that of which the resistance had been taken, as we could not be sure that the closeness of contact between the mercury and the glass was always the same, so that the same volume of mercury would always be contained in the same length of tube, nor that the tube itself was in no way altered by the action of the caustic potash used to clean it. The plan adopted was, therefore, after measuring the resistance, to keep the tube horizontal so as to retain in it most of the mercury while the terminals were removed, and then with microscopes and divided rod to measure the thread of mercury in the same way as the tubes were measured. The length so obtained is called in the table  $l$ . The greatest difference between  $l$  and  $L$  (that in measurement 11) is scarcely over 1 per cent., and in most cases the difference is considerably less, so that, considering how nearly cylindrical the tubes were, the error in the mean section introduced by using a thread of length  $l$  instead of  $L$  is quite inappreciable. It was another advantage of our method that it avoided the necessity of filling the tube under mercury, which it would have been difficult to do with a tube so long as IV.

The only difficulty in measuring the thread of mercury arose from the convexity of its ends. This was overcome by pressing them flat with little flat-ended vulcanite pins made to fit into the tube. The curvature of the ends when free was not always the same; but it was found that the length of the mercury held with pins varied little from the number calculated on the assumption that the ends were hemispherical, namely, the length of the portion of the column of mercury which was in contact with the glass added to two-thirds of the difference between this length and that between the convex extremities. In some cases, where, owing to the pins not fitting very well or other causes, there was a difficulty in flattening the ends properly, the calculated value was

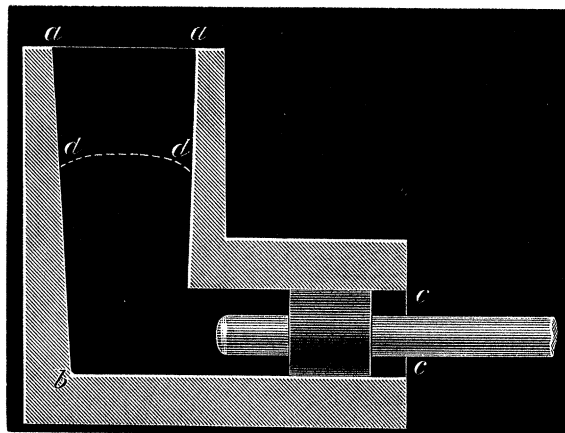


used. A thermometer lay beside the tube during the measurement, so as to give the temperature  $t$ . After the measurement, the mercury was blown out into a small crucible and weighed. Care had to be taken not to leave behind minute globules, which, owing probably to the small portion of the tube unoccupied by mercury during the measuring becoming damp from the air of the room or from the fingers, tended to adhere to the glass near the ends.

In three cases (No. 5 as above mentioned and Nos. 3 and 9) the mercury weighed and measured was not that of which the resistance was taken.

No. 3 was done before it occurred to us that there might be an advantage in carrying out both operations with the same filling, and in No. 9 about one-tenth of the mercury was spilt accidentally and had to be replaced.

The equality of the arms of the balance used for the weighing was tested. The weights were compared among themselves and found to be free from appreciable error.



The terminals were composed of L-shaped pieces of ebonite, hollowed out in the manner shown (about full-size) in the figure. Each end of the tube was furnished with a short length of thick rubber tubing, by which the aperture between the glass and the ebonite was closed air-tight. As a further precaution, the space at  $c c$  beyond the rubber was filled up by pouring in melted paraffine wax.

After the terminals were fitted the tube was again aspirated with dry air through tubes in corks inserted at  $a a$ , and then filled with mercury, which was poured in to one terminal and allowed to run slowly through to the other till it stood at a considerable height, represented by  $d d$ , in both terminals. The tube was then placed in a wooden trough and covered with ice. Our reason for using vulcanite terminals rather than glass ones was the fear that under the influence of the ice moisture would collect on the portion of glass above the mercury and serve as a conductor. We certainly avoided all difficulty of this kind by using vulcanite. On the other hand, we probably increased a difficulty which would have existed in any case, namely, that of getting the temperature of the portion of the tube which was within the terminal

down to  $0^{\circ}$ . This portion of the tube was about 2 centims. at each end, or about 5 per cent. of the length in the case of tube I., and about 2 per cent. in the case of tube IV. What the exact temperature of this part of the tube was it is impossible to say, but it was ascertained that the temperature of the mercury in the terminals with the copper connecting rods *in situ* was not higher than  $5^{\circ}$  or  $6^{\circ}$ , depending in some degree on the extent to which the ice was piled up round the cup. The mean temperature of the parts of the tubes not directly exposed to ice can hardly have been so high as  $2^{\circ}$ . Supposing it to have been  $2^{\circ}$ , and taking the case of tube I., where the largest proportion of the whole length was within the terminals, the effect would be an overestimate of  $r$  by about  $\cdot 00008$ . In the case of tube IV. the error in  $r$  would be less than the half of this.

The tubes were connected with the resistance balance by copper rods, well amalgamated, of which one end stood on the bottom of the vulcanite terminals, so that a considerable portion of the amalgamated copper surface was in contact with the mercury. The rods were kept at a little distance from the ends of the tubes. Dr. MATTHIESSEN brought flattened copper rods up against the ends of his tubes, but this plan appeared open to objection, since it would be very difficult to secure complete contact between the copper and glass all round the edge of the orifice, especially under an opaque fluid like mercury; and any defect in such contact would render necessary an unknown correction. We preferred, therefore, to let the ends of the tube open without obstruction into the mercury cup, which may be regarded as of infinite extent by comparison. The correction necessary to take account of the resistance of the mercury beyond the ends of the tube has already been considered.

The resistance of the rods used to connect I., II., and III. with the bridge was about  $\cdot 00215$  B.A. unit. With tube IV. an additional rod had to be introduced to get the necessary length. This brought the resistance of the rods up to  $\cdot 00291$ . The other end of the rods fitted into mercury cups on the resistance balance.

The balance used was one designed by Professor FLEMING (Phil. Mag., ix., p. 109, 1880), in which Professor CAREY FOSTER's method is employed of interchanging the resistances in the two arms of the balance containing the graduated wire, so that the difference between them is expressed in terms of the wire. One thousand divisions of the graduated wire are stated by Professor FLEMING to equal  $\cdot 0498$  B.A. unit, and experiments of our own also showed it to be about  $\cdot 05$ . The wire is of platinum-iridium, and as it has a high temperature coefficient compared with the platinum-silver of the standard coils, we thought it undesirable to use much over 100 divisions of it. In order to avoid this in the case of tubes I. and IV. it was necessary to introduce coils from a resistance box in multiple arc. The resistance box employed was one by Messrs. ELLIOTT Brothers. With tube I., 20 ohms from the box were used in multiple arc with the standards against which the tube was balanced, and in the case of tube IV. 24 ohms were used in multiple arc with the tube itself. Tubes II. and III. were balanced against the standard coil belonging to the British Association and deposited

at the Cavendish Laboratory, called *F*. For tube IV. another of their unit coils, called the *Flat coil*, was used in multiple arc with *F*. For tube I., *F* and a five-ohm coil were used in multiple arc. The standard coils belonging to the British Association have recently been carefully compared with each other by Professor FLEMING, who has drawn out a chart in which is recorded their variation with temperature, together with their resistance in terms of the mean of their resistances at the temperatures at which they were originally considered to be correct. The values of *F* and of the *Flat coil*—both platinum-silver coils—were taken from this chart. The five-ohm coil had been compared with the British Association standards by ourselves. It was also of platinum-silver, and its temperature coefficient was assumed to be the same as that of the others.

The standard coils were immersed in water whose temperature was observed each time a resistance was measured. These temperatures are given in the table. It may be worth remarking that the resistances were taken in a different room from that in which the lengths were measured, which accounts for the difference between *t* and the temperature of the standards. The thermometer used to find all the temperatures was graduated to fifths, and was corrected by one which had been verified at Kew.

When one coil only was used to balance the tube, its terminals fitted directly into the mercury cups of the bridge, but when two were used in multiple arc their terminals were put into larger mercury cups, which were connected with the mercury cups of the bridge by short copper connecting pieces of about .00017 ohm resistance.

All the measurements were repeated with reversed battery currents, in order to eliminate thermoelectric disturbance. The readings with battery current each way usually agreed very closely, and the mean of the two was adopted.

It will be observed that the values of *R* for tube IV. differ by nearly two parts in 10,000, and that there is a less proportional difference, but still an appreciable one, for the other tubes. The greatest actual difference between any two of the values in the table for the same tube is .00014 ohm. Some small error is due to neglect of the change of resistance of the copper connecting rods and of the bridge wire with temperature. A change of 4° in the temperature of the rods would make a difference of about .00003 ohm. There is further a probability of error in ascertaining the temperature of the standard coil. A difference of  $\frac{1}{10}^{\circ}$  in this also introduces a difference of .00003 ohm in the resistance; and there is not only a probable error of perhaps  $\frac{1}{10}$  in finding the temperature of the water in which the coil is immersed, but there is no certainty that the coil follows the water exactly. There is evidence, however, that the differences in *R* are partly due to a real difference in the resistance of different fillings of the tube—whether owing to microscopic bubbles or to a thin varying layer of air between the mercury and the glass, or to what cause, we were unable to determine.\*

\* A variation in the closeness of contact between mercury and glass amounting to less than one-fifth of a wave-length of mean light would account for the difference of resistances in the two fillings of tube IV.

We found some reason for thinking that the resistance tended to diminish with time when the mercury remained long in the tube. To examine this we filled tube II. on April 3rd, and found its resistance to be  $\cdot99077$ . It was then left standing full of mercury till April 18th, when the resistance was  $\cdot99055$ . This difference can hardly be relied upon; and in any case the experiments we have tabulated cannot well be affected by any change of this kind, as the interval between the measurement of resistance and that of volume was very short, except in cases 1 and 7. In case 7 the tube stood full of mercury for two days after the resistance was taken. In case 1 the resistance was measured on two successive days, and the mean of the two values taken. The second was the lowest by  $\cdot00020$ , possibly owing to an error. The length was measured immediately after the last measurement of resistance.

The variations in the values of  $r$  are, as we should expect, greater than those in  $R$ , being affected by probable errors in the other data. The extreme difference amounts to less than 6 in 10,000, and the greatest divergence from the mean value is 3·3 in 10,000.

The mean value of  $r$  according to these experiments,  $\cdot95418$ , lies between that deduced from Dr. SIEMENS' experiments for his 1864 standard, namely,  $\cdot9534$ , and Dr. MATTHIESSEN's value, namely,  $\cdot9619$  (Phil. Mag., May, 1865), but the difference between our value and Dr. MATTHIESSEN's, namely,  $\cdot00772$ , is nearly ten times as great as that between ours and Dr. SIEMENS'. We are unable to account satisfactorily for this large difference. One point, however, is worth noting. Dr. MATTHIESSEN measured the resistance of the mercury in his tubes, not at zero, but at temperatures between  $18^{\circ}$  and  $19^{\circ}\cdot1$  (Report of British Association Committee for 1864). To deduce the specific resistance at zero, therefore, he must have assumed the coefficient of variation with temperature, and presumably—though it is nowhere stated in the Report—he used that found from his own experiments (Phil. Trans., 1862), namely,  $\cdot074^*$  per cent. per degree. Our own observations have led us to suspect that this value is too small. We made three comparisons of the resistance of tube III. in ice, and in water at approximately the temperature of the room, and one similar comparison with tube IV. The results are given in the following table. Our arrangements were not adapted for observing the resistance at other temperatures, as the open trough afforded no means of checking rapid change.

\* This is the value which results from the experiments made at  $0^{\circ}$  and at about  $20^{\circ}$ .

Date.	No. of tube.	Mean temperature of water in the trough.	Resistance in water.	Resistance at 0°.	Difference for 1° ÷ resistance at 0°.	Mean of the four values in the last column.
March 13 . . .	III.	12·7	1·00814	·99720	·000863	·000861*
„ 14 . . .	III.	13·25	1·00874	·99725	·000870	
„ 28 . . .	III.	12·8	1·00810	·99720	·000854	
„ 24 . . .	IV.	12·5	·51318	·50774	·000857	

The above determined mean coincides with the value found by SCHRÖDER VAN DER KOLK,† whose observations, however, related to a much greater range of temperature. An observation by WERNER SIEMENS‡ between the temperature 18°·5 and 0° gives for the coefficient ·00090.

The difference between the coefficients ·00074 and ·00086, as applied to the reduction from 18°·7 (the mean temperature of the tubes in Dr. MATTHIESSEN'S observations) to 0°, would account for about one quarter of the difference between his results and our own.

The remainder of the discrepancy may possibly be connected with the manner in which Dr. MATTHIESSEN'S tubes were calibrated. Although in the description of the process a *small* column of mercury is spoken of (Reprint, p. 128), it is distinctly stated on the preceding page that the lengths of the columns of mercury were 383, 291, 245 millims. respectively, *i.e.*, nearly half the lengths of the tubes. It is possible that this may be a mistake; but if such lengths were really used, the correction for conicality would have been much underestimated, so that the specific resistance of mercury would come out too high. In the case of uniform conicality the true correction would be four times as great as that obtained by applying the formula applicable to short threads, to cases where the length is about half that of the tube.

[*January*, 1883.—The measuring rod and the weights used in the above investigation have been compared with standards verified by the Board of Trade, and the errors have been found to be negligible. But since the value of  $\rho$  employed relates to weighings *in vacuo*, a corresponding correction is called for here. On this account the final number, ·95418, should be reduced to

·95412.]

\* It should be noticed that the resistances here compared are those of the contents of a certain glass tube at various temperatures, so that the accompanying temperature variations of length and section are determined by the properties of glass and not by the properties of mercury. The results are therefore not quite comparable with those obtained in similar experiments with solid metallic wires, which are free to determine for themselves their length and section.

† Pogg. Ann., cx., 1860.

‡ Ibid., cxiii., 1861.